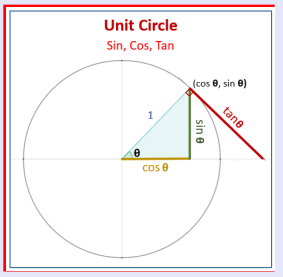
Trigonometry

Lecture 29



Feb 19-8:47 AM

1) Convert 40° to radians

$180^\circ = \pi$ Rad.

$1^\circ = \frac{\pi}{180}$ Rad.

$40^\circ = \frac{40\pi}{180}$ Rad.

$= \frac{2\pi}{9}$ Rad.

$\approx .698$ Rad.

2) Convert $\frac{5\pi}{8}$ to degrees.

π Rad $= 180^\circ$

1 Rad $= \left(\frac{180}{\pi}\right)^\circ$

$\frac{5\pi}{8}$ Rad $= \left(\frac{5\pi}{8} \cdot \frac{180}{\pi}\right)^\circ = \frac{5 \cdot 180}{8}^\circ$

$= 112.5^\circ$

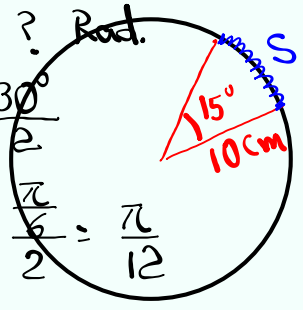
Oct 17-10:31 AM

Consider a Circular sector with Central angle of 15° and 10cm Radius

$15° = ?$ ~~Rad.~~

$15° = \frac{30°}{2}$

$= \frac{\pi}{6} \div 2 = \frac{\pi}{12}$



1) Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(10)^2 \cdot \frac{\pi}{12} = \frac{100\pi}{2 \cdot 12}$ (25, 3)

$= \boxed{\frac{25\pi}{6} \text{ cm}^2}$

2) Arc length

$S = r\theta = 10 \cdot \frac{\pi}{12} = \boxed{\frac{5\pi}{6} \text{ cm}}$

Oct 17-10:35 AM

A fan has a blade of $r = 3$ 3-ft long.

It makes $12 \cdot 2\pi = 24\pi$ 12 revolutions per second.

Find Linear and angular Speed

$v = \frac{s}{t}$ $\quad w = \frac{\theta}{t}$ $\quad v = rw$

$v = rw$

$= 3 \cdot 24\pi = 72\pi$

≈ 226 ft/sec.

$w = \frac{24\pi}{1}$ Rad/sec

$= 24\pi$ Rad./sec.

Oct 17-10:40 AM

A circular sector has a central angle of 120° and area of 54.8 cm^2.

Find its radius.

$A = \frac{1}{2} r^2 \theta$

$54.8 = \frac{1}{2} \cdot r^2 \cdot \frac{2\pi}{3}$

$3(54.8) = r^2 \cdot \pi$

$\frac{3(54.8)}{\pi} = r^2$

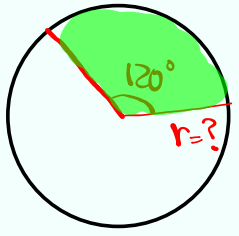
$r^2 \approx 52.330$

$r \approx \sqrt{52.330}$

$r \approx 7.2$ cm

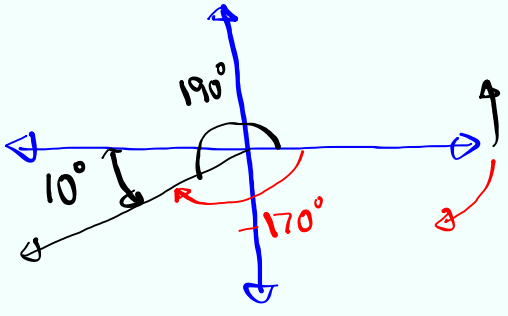
$120° = 2(60°)$

$= \frac{2\pi}{3}$



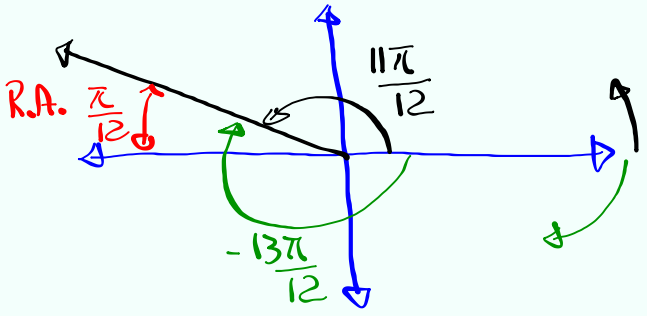
Oct 17-10:45 AM

Draw 190° & −170°, give the Ref. angle for each.



Oct 17-10:50 AM

Draw $\frac{11\pi}{12}$ & $-\frac{13\pi}{12}$, give Ref. angle.



Oct 17-10:52 AM

A Small pulley is turning 145 Revolutions Per minute

$145(2\pi) = 290\pi$ Radians

Find angular Speed per Second.

$$\omega = \frac{\theta}{t} = \frac{290\pi}{1\ \text{Min.}} \cdot \frac{1\ \text{Min.}}{60\ \text{Sec.}}$$

$$= \frac{290\pi}{60}\ \text{Rad/Sec.}$$

$$\approx 15.2\ \text{Rad./Sec.}$$

Oct 17-10:55 AM

$10(2\pi) = 20\pi$

A 3-ft blade makes 10 Revolutions in 42 Seconds.

Find angular speed rad/min.

$$\omega = \frac{\theta}{t} = \frac{20\pi \text{ Rad}}{42 \text{ Seconds}} \cdot \frac{60 \text{ Seconds}}{1 \text{ Minute}}$$

$$= \frac{20\pi \cdot 60}{42} \text{ Rad/min.}$$

$$\approx 90 \text{ Rad./min.}$$

Find Linear Speed.

$v = r\omega$ $\quad v = 3 \cdot 90$

$= 270$ ft/min.

Oct 17-10:59 AM

$\tan\theta = -\frac{1}{5}$

Find

1) $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{1}{5}} = -5$

2) $\tan(-\theta) = -\tan\theta = -\left(-\frac{1}{5}\right) = \frac{1}{5}$

only $\cos(-\theta) = \cos\theta$

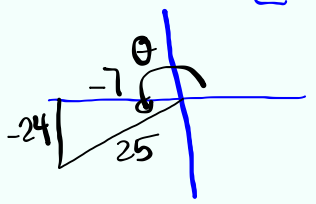
$\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$

Oct 17-11:05 AM

Complete the chart below if $\pi < \theta < \frac{3\pi}{2}$

$\sin\theta = -\frac{24}{25}$	$\csc\theta = -\frac{25}{24}$
$\cos\theta = -\frac{7}{25}$	$\sec\theta = -\frac{25}{7}$
$\tan\theta = \frac{24}{7}$	$\cot\theta = \frac{7}{24}$



$\sqrt{25^2 - (-24)^2} = 7$

Oct 17-11:08 AM

Simplify $\frac{1}{1 - \sec x} + \frac{1}{1 + \sec x}$

$= \frac{1 + \sec x + 1 - \sec x}{\underbrace{(1 - \sec x)(1 + \sec x)}_{\text{Foil}}}$

$= \frac{2}{1 - \sec^2 x} = \frac{2}{1 - 1 - \tan^2 x}$

$= -2\cot^2 x$

Recall

$1 + \tan^2 x = \sec^2 x$

Oct 17-11:12 AM

Verify

$$\frac{1+\sin x+\cos x}{1-\sin x+\cos x} = \frac{1+\sin x}{\cos x}$$

$$\frac{1+\sin x+\cos x}{1-\sin x+\cos x} \stackrel{?}{=} \frac{1+\sin x}{\cos x}$$

Cross-multiply

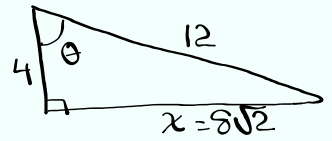
$$\cos x(1+\sin x+\cos x) \stackrel{?}{=} (1+\sin x)(1-\sin x+\cos x)$$

$$\cos x+\cos x\sin x+\cos^2 x \stackrel{?}{=} 1-\sin x+\cos x+\sin x-\sin^2 x+\sin x\cos x$$

$$\cos x+\cos x\sin x+\cos^2 x \stackrel{?}{=} \cos x+\sin x\cos x+\cos^2 x$$

Oct 17-11:16 AM

Use drawing below to find



$4^2+x^2=12^2$

$16+x^2=144$

$x^2=128$

$x=\sqrt{128}$

$x=\sqrt{4\cdot 32}=2\sqrt{16\cdot 2}=8\sqrt{2}$

$\sin\theta = \frac{8\sqrt{2}}{12} = \frac{2\sqrt{2}}{3}$

$\sin\frac{\theta}{2}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{4}{12}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2$

$= \frac{1}{9} - \frac{8}{9}$

$= \boxed{\frac{-7}{9}}$

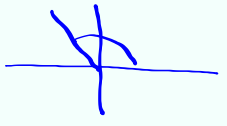
$\tan\frac{\theta}{2}$

$\cos\theta = \frac{4}{12}$

$\cos\theta = \frac{1}{3}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right) \approx 71°$

$2\theta \approx 142°$



Oct 17-11:26 AM

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\frac{1}{3}}{2}}$

$= \sqrt{\frac{3-1}{6}} = \sqrt{\frac{2}{6}}$

$= \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$0° < \theta < 90°$

$\theta \approx 71°$

$\frac{\theta}{2} \approx 35.5°$

QI

$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

$\tan\theta = \frac{\frac{2\sqrt{2}}{3}}{1+\frac{1}{3}} = \frac{2\sqrt{2}}{3+1} = \frac{2\sqrt{2}}{4} = \boxed{\frac{\sqrt{2}}{2}}$

LCD = 3

Oct 17-11:33 AM

Write as product

$\sin 4x + \sin 6x$

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$= 2\sin\frac{4x+6x}{2}\cos\frac{4x-6x}{2}$

$= 2\sin 5x \cos(-x)$

$= 2\sin 5x \cdot \cos x$

Oct 17-11:38 AM

Write as sum

$$\sin 5x \sin x = \frac{1}{2}\left[\cos(5x-x) - \cos(5x+x)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$= \frac{1}{2}\left[\cos 4x - \cos 6x\right]$$

Oct 17-11:41 AM